Title: What About Medians?

Brief Overview:

The students will be able to draw a triangle and the three medians of a triangle using Geometer's Sketchpad. The students will be able to identify and measure the medians and the segments of the median formed by the vertices to the centroid and the centroid to the midpoint of the opposite side using Geometer's Sketchpad. The students will be able to draw an important conclusion about the relationships among those segments.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*

- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*

- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*

- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*

- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

  Number and Operations
  The students will be able to understand numbers, ways of representing numbers, relationships among numbers, and number systems.

  Geometry
  The students will be able to analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships. They will be able to use visualization, spatial reasoning, and geometric modeling to solve problems.

<u>Measurement</u>
The students will be able to apply appropriate techniques, tools, and formulas to determine measurements.

- **Process Standards**

<u>Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation</u>
These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

The students will be able to use Geometer's Sketchpad to demonstrate and state the relationship that exists between the two segments of the median formed with the centroid and the relationship that exists between the segment from the vertex to the centroid and the median.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra
- **1.1.1**
The students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

Geometry, Measurement, and Reasoning
- **2.1.1**
The students will describe the characteristics of geometric figures and will construct or draw geometric figures using technology as tools.

- **2.1.2**
The students will identify and verify properties of geometric figures using concepts from algebra and using the coordinate plane.

- **2.1.4**
The students will validate properties of geometric figures using appropriate tools and technology.

- **2.3.1**
The students will use algebraic and geometric properties to measure indirectly.

- **2.3.2**
The students will use techniques of measurement and will estimate, calculate, and compare perimeter, circumference, area, volume, and surface area of two- and three-dimensional figures and their parts. The results will be expressed with appropriate precision.

Grade/Level:

Grades 9 and 10 Geometry; Enrichment Potential for Grades 6 to 8

Duration/Length:

Two or three class periods, approximately 45 minutes in length

Prerequisite Knowledge:

The students should have working knowledge of the following skills:

- Using Geometer's Sketchpad to measure the lengths of segments and the areas of polygonal regions
- Comparing values to determine whether they form a constant ratio
- Graphing points on a coordinate grid
- Determining the midpoint of a segment for which the coordinates of the endpoint are known
- Using triangles and the terms needed to classify them
- Using Geometer's Sketchpad to construct simple figures consisting of points and segments and polygonal regions
- Recognizing the special points and segments related to the triangle
- Computing the area of a triangle using the formula $A = \frac{1}{2}bh$. (extension activity)

Student Outcomes:

The students will:

- use Geometer's Sketchpad to construct a diagram in which a triangle, its medians, and the centroid are drawn.
- use Geometer's Sketchpad to determine the lengths of the median and the two segments formed using the centroid with the vertex and the centroid with the midpoint of the opposite side.
- use several triangles to compare the measurements of the three segments for each median; draw the conclusion that the ratio of the segment from vertex to centroid with the segment from centroid to midpoint is two to one and the conclusion that the medians of a triangle intersect at a point that is two-thirds of the way from the vertex to the midpoint of the opposite side.
- use Geometer's Sketchpad to plot vertices of a triangle at lattice points, reconstruct the centroid, and determine its coordinates.
- examine the coordinates of the three points to observe that the coordinates of the centroid are the averages of the coordinates of the vertices.

- use Geometer’s Sketchpad to construct polygonal regions and determine their areas. Compare the areas of the six small triangles formed by a vertex, a midpoint on an adjacent side and the centroid with one another and the area of the original triangle, and observe that each small triangle has the same area and that each small triangle’s area is one-sixth of the area of the original triangle. (This is an outcome for students who perform the extension exercises.)

Materials/Resources/Printed Materials:

- Geometer’s Sketchpad
- Geometer’s Sketchpad Instruction Sheets
- Student Activity Sheets
- Graph paper, straight edge, ruler
- Student Assessment Sheet

Development/Procedures:

The students will use Geometer’s Sketchpad to construct a triangle and its medians. They will use Geometer’s Sketchpad to measure the lengths of the median, the segment from the vertex to the centroid, and the segment from the centroid to the midpoint. They also will compare these lengths for all three medians and with several triangles. The students will draw the conclusion that the centroid is located on each median two-thirds of the distance from the vertex to the midpoint of the opposite side.

Extension/Follow Up:

Continuing the investigation of relationships that are found using the centroid, students might investigate the areas of the triangular regions formed using vertices and the centroid (three smaller triangles within the original triangle) to discover (and even perhaps prove) that the areas of these smaller triangles are all equal to each other and thus each is one-third of the original area. There are also six small triangles formed each having as its vertices a vertex of the original triangle, the midpoint of an adjacent side and the centroid. Investigating these areas, students would find that these triangles all have equal area and thus each is one-sixth of the area of the original triangle. This too can be proved.

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- **Assessment Content Standards**

<u>Number and Operation</u>
The students will be able to understand numbers, ways of representing numbers, relationships among numbers, and number systems.

<u>Geometry</u>
The students will be able to analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships. They will be able to use visualization, spatial reasoning, and geometric modeling to solve problems.

<u>Measurement</u>
The students will be able to apply appropriate techniques, tools, and formulas to determine measurements.

Teacher's Guide

The student activity sheet is comprehensive in its description of how students will proceed. If Geometer's Sketchpad is used in the classroom on a regular basis, no orientation to the software would be needed. For classes in which Geometer's Sketchpad is not well known, a teacher-directed overhead demonstration in a lab setting would be recommended. Teacher would use the Student/Teacher Instruction Sheet for Geometer's Sketchpad showing the techniques needed to perform the requested constructions. Ideally, students will have a copy of the instructions and will perform the same processes at their own stations during the time the teacher is demonstrating.

Administering the Assessment

One 45 minute session should be allowed for the assessment. Students should be at workstations in a computer lab equipped with Geometer's Sketchpad. Students should also be provided with graph paper and ruler. The assessment activity may be performed with the software or as a paper-and-pencil activity (or both). Students will attach either a computer printout of their constructions and measurements or their graph paper drawing with measurements of the relevant segments written down.

For a complete answer to the constructed response on which the score will be based, students will have to support their conclusions with numeric values that confirm their statements.

Assessment

Scoring Guide: Rubric for "What About Medians?" Assessment

4: The student correctly constructs a triangle with three medians. The student gives the lengths of the medians and the lengths from a vertex to the centroid for each median. The student also gives the length from the centroid to the opposite side for each median. The student clearly states the relationship that exists between the lengths of the medians and the lengths of the segments formed with the vertex to centroid.

3: The student correctly constructs a triangle with three medians. The student gives the lengths of the medians and the lengths from a vertex to the centroid for each median. The student also gives the length from the centroid to the opposite side for each median. The student states, with minor errors, the relationship that exists between the lengths of the medians and the lengths of the segments formed with the vertex to centroid.

2: The student correctly constructs a triangle with three medians. The student attempts to identify the medians and the centroid. The student attempts to give the lengths of the medians and the lengths from a vertex to the centroid for each median. The student states, with major errors, the relationship that exists between the lengths of the medians and the lengths of the segments formed with the vertex to centroid.

1: The student correctly constructs a triangle and attempts to construct the medians. The student attempts to give the length of the medians and the length of the segment from vertex to centroid. The student does not state the relationship that exists between the lengths of the medians and the lengths of the segments formed with the vertex to centroid.

0: The student does not attempt to solve the problem.

USING GEOMETER'S SKETCHPAD
STUDENT/TEACHER INSTRUCTION SHEET

Note that in general you will want the "arrow" (select) tool to be highlighted except when you are drawing or labeling objects (points, segments, circles, etc.)

To draw a triangle

1. Using the "point" tool and holding down the "shift" key, plot three points.
2. On the "construct" menu, click on "segment"
3. Using the "hand" tool, label the points (Note: GSP will automatically assign letter names A, B, C, etc alphabetically in the order that you plotted them.)

To change names of points or segments

(Note that GSP will automatically assign letter names alphabetically to points in the order of their creation. Sometimes you may prefer a different naming system)

1. Once points (segments) have been named, using the "hand" tool, align the hand on the point so the hand is transparent and contains the letter "A"
2. Double click and enter the new name when the dialogue box comes up.
3. Drag labels, if you want, to position them convenient to the objects they describe.

To measure lengths of segments (two methods)

On the "display" menu, click on "preferences". Under "distance units", choose either "inches" or "cm" as instructed by your teacher. Under "precision" select "thousandths".

A. By selecting segments

    1. Select the segment(s) you wish to measure (for multiple segments, hold down the "shift" key)
    2. On the "measure" menu, click on "length".

B. By selecting endpoints (only one segment can be measured at a time using this method)

    1. Holding down the "shift" key, select (in the order you want the segment to be named) the two endpoints.
    2. On the "measure" menu, click on "distance".

To construct a table of measurement values

Be certain that no part of the diagram is selected.

1. Holding the "shift" key down, select the measures (already computed and printed on the screen) to be put in the table.
2. On the "measure" menu, click on "tabulate". A table should appear on the screen. If its position is inconvenient, drag it to a more appropriate location.
3. To add a column of additional values for the set of measures of the same parts of a diagram (altered, for example, by dragging points to new locations), double click on the numerical entries column.

To construct medians and centroid

1. Select the sides on which you want to construct midpoints.
2. On the "construct" menu, click on "point at midpoint".
3. Click the mouse in an empty part of the screen (to turn off selections or new creations).
4. Holding down the "shift" key, select one vertex and its corresponding midpoint and construct the segment (as above). Click the mouse and then repeat this. Construct all three medians of the triangle using this method.
5. Holding down the "shift" key, select two of the medians.
6. On the "construct" menu, click on "point at intersection".

To construct a diagram using coordinate graphing techniques

1. On the "graph" menu, click on "show grid". Notice the point at (1,0). Selecting and dragging this point allows you to adjust the size of your graph. Using the "point" tool, you may plot the points at the specified coordinates by hand. OR
2. On the "graph" menu, click on "plot points". Before plotting any points, select "free points".
3. Still in the "plot points" dialogue box, enter the x-coordinate of the first point to be plotted, push the "tab" key and enter the y-coordinate. Click on "add". Enter additional points the same way. When all of the points are entered, click "plot".
4. To see a list of the coordinates of the points, select the points (holding down the "shift" key to select multiple points), and on the "measure" menu, click on "coordinates" and you will get a list of the points and their coordinates. (Be sure you have named the points so you can tell which is which).

NAME ____________________________
DATE______________________________
TEACHER__________________________

WHAT ABOUT MEDIANS?

Using Geometer's Sketchpad, construct acute ΔABC. Then construct all three medians for the triangle. Recall that the intersection of the medians is called the *centroid*. Label the midpoint of $\overline{AC}$ as X, the midpoint of $\overline{AB}$ as Y, and the midpoint of $\overline{BC}$ as Z. Label the centroid as T.

Use the measurement facility of Geometer's Sketchpad to determine each of the following lengths and fill in the table on page 2 for Triangle 1. You may want to select all of the measurements after you compute them and have Geometer's Sketchpad construct a table of these values right there on your screen. Print out a copy of this screen to include with this worksheet.

Redo this experiment with an obtuse triangle (Notice that you can simply "drag" one or more of the vertices in your original triangle until you get a new triangle that has the right shape. The advantage of doing this is that you don't have to reconstruct all of the medians, and as you change the triangle, Geometer's Sketchpad automatically repositions all the other constructs that you have already added as well as recomputing the measures of the segment lengths that you have requested). Print out a copy of this screen to include with this worksheet.

Redo this experiment once more with any triangle of your choice (you may want to use a special triangle such as an equilateral, isosceles, or right triangle—do you know how to construct a triangle to these specifications?) Print out a copy of this screen to include with this worksheet.

If you double click on the table of values that Geometer's Sketchpad has produced, an additional column will be placed to the right of the previous set of values reflecting the new values for the new triangle. You can easily copy these sets of values into the table below.

	AT	TZ	AZ	CT	TY	CY	BT	TX	BX
Triangle 1									
Triangle 2									
Triangle 3									

Look at the sets of values in the first three columns. Do you see any special relationships among the lengths of the segments?

Investigate the sets of values in the next three columns. Then check to see whether the same relationships exist for the values in the last three columns. Explain, using appropriate terms, all the relationships you have found.

Using the “graph” menu in Geometer’s Sketchpad, plot the following points as the vertices of a triangle. $A(3,-3), B(7,5), C(-1,-5)$ As you have done in the previous activity, construct the triangle, its three medians, and the centroid. Using the “measure” menu, determine the coordinates of the centroid. What is the relationship between the coordinates of the vertices and the coordinates of the centroid?

Confirm your hypothesis by redoing the above graphing experiment with another set of points. Choose your own. You may want to draw a triangle on a piece of graph paper and then use the coordinates of the points you chose in Geometer’s Sketchpad. Print out a copy of the Geometer's Sketchpad screen for your triangle and include it with this worksheet.

The centroid is also called the “center of mass”. It is the single point on which the triangle would ideally “balance” if we laid a model of the triangle on, for example, a pencil point. Relate the mathematical idea of the location of the centroid to this physical fact.

TEACHER KEY

WHAT ABOUT MEDIANS?

Using Geometer's Sketchpad, construct acute ΔABC. Then construct all three medians for the triangle. Recall that the intersection of the medians is called the *centroid*. Label the midpoint of $\overline{AC}$ as X, the midpoint of $\overline{AB}$ as Y, and the midpoint of $\overline{BC}$ as Z. Label the centroid as T.

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Redo this experiment once more with any triangle of your choice (you may want to use a special triangle such as an equilateral, isosceles, or right triangle—do you know how to construct a triangle to these specifications?) Print out a copy of this screen to include with this worksheet.

If you double click on the table of values that Geometer's Sketchpad has produced, an additional column will be placed to the right of the previous set of values reflecting the new values for the new triangle. You can easily copy these sets of values into the table below.

(Answers will vary depending on the particular triangle that students draw. General answer patterns will be given for one triangle below)

	AT	TZ	AZ	CT	TY	CY	BT	TX	BX
Triangle 1	***2a***	***a***	***3a***	***2c***	***c***	***3c***	***2b***	***b***	***3b***
Triangle 2									
Triangle 3									

Look at the sets of values in the first three columns. Do you see any special relationships among the lengths of the segments?

The segment from the vertex to the centroid is twice as long as the segment from the centroid to the midpoint of the opposite side. The centroid is two-thirds of the way along the median.

Investigate the sets of values in the next three columns. Then check to see whether the same relationships exist for the values in the last three columns. Explain, using appropriate terms, all the relationships you have found.

The same relationship holds for the segments formed by the centroid on the other two medians. All three medians intersect at a single point and that point is two-thirds of the way from the vertex to the opposite side.

Using the “graph” menu in Geometer’s Sketchpad, plot the following points as the vertices of a triangle. $A(3,-3), B(7,5), C(-1,-5)$ As you have done in the previous activity, construct the triangle, its three medians, and the centroid. Using the “measure” menu, determine the coordinates of the centroid. What is the relationship between the coordinates of the vertices and the coordinates of the centroid?

Geometer's Sketchpad gives the coordinates of the centroid as (3,-1). The x-coordinate of the centroid is the same as the average of the x-coordinates of the vertices. The y-coordinate of the centroid is the same as the average of the y-coordinates of the vertices.

Confirm your hypothesis by redoing the above graphing experiment with another set of points. Choose your own. You may want to draw a triangle on a piece of graph paper and then use the coordinates of the points you chose in Geometer’s Sketchpad. Print out a copy of the Geometer's Sketchpad screen for your triangle and include it with this worksheet.

The centroid is also called the “center of mass”. It is the single point on which the triangle would ideally “balance” if we laid a model of the triangle on, for example, a pencil point. Relate the mathematical idea of the location of the centroid to this physical fact.

It makes sense that a point that is in an "average" position with respect to the vertices of the figure (probably this only would work for convex polygons) would be the point for which a natural "balance" would occur. In convex polygons with more than three sides, the center of mass would also be located at a point inside the polygonal region that has coordinates that average the coordinates of the vertices.

NAME ______________________
DATE ______________________
TEACHER ___________________

PERFORMANCE ASSESSMENT

WHAT ABOUT MEDIANS?

DIRECTIONS: Using the graph menu in Geometer's Sketchpad (or a piece of graph paper) plot the following points $C(6,-1)$, $D(-2,-2)$, and $E(5,-6)$ as the vertices of a triangle. Label the midpoint of $\overline{CD}$ as H, the midpoint of $\overline{CE}$ as F and the midpoint of $\overline{DE}$ as G. Label the centroid as I. Determine the measurements of all the requested segments. If you use Geometer's Sketchpad to construct the triangle, you must print out the screen on which your work is done and include it with this assessment.. If you use a piece of graph paper, you are to include it when you turn in the assessment

Refer to your triangle to complete the following. (If possible, give the segment measures accurate to the thousandths place)

1. CG = ________________ inches
2. DF = ________________ inches
3. EH = ________________ inches
4. CI = ________________ inches
5. DI = ________________ inches
6. EI = ________________ inches
7. IG = ________________ inches
8. IF = ________________ inches
9. IH = ________________ inches

10. Name the coordinates for the centroid. _________________

11. Explain the special relationship that exists between the length of the median and the location of the centroid. Support your statements by using examples from the triangle and its values above.

TEACHER _____*KEY*__________

PERFORMANCE ASSESSMENT

WHAT ABOUT MEDIANS?

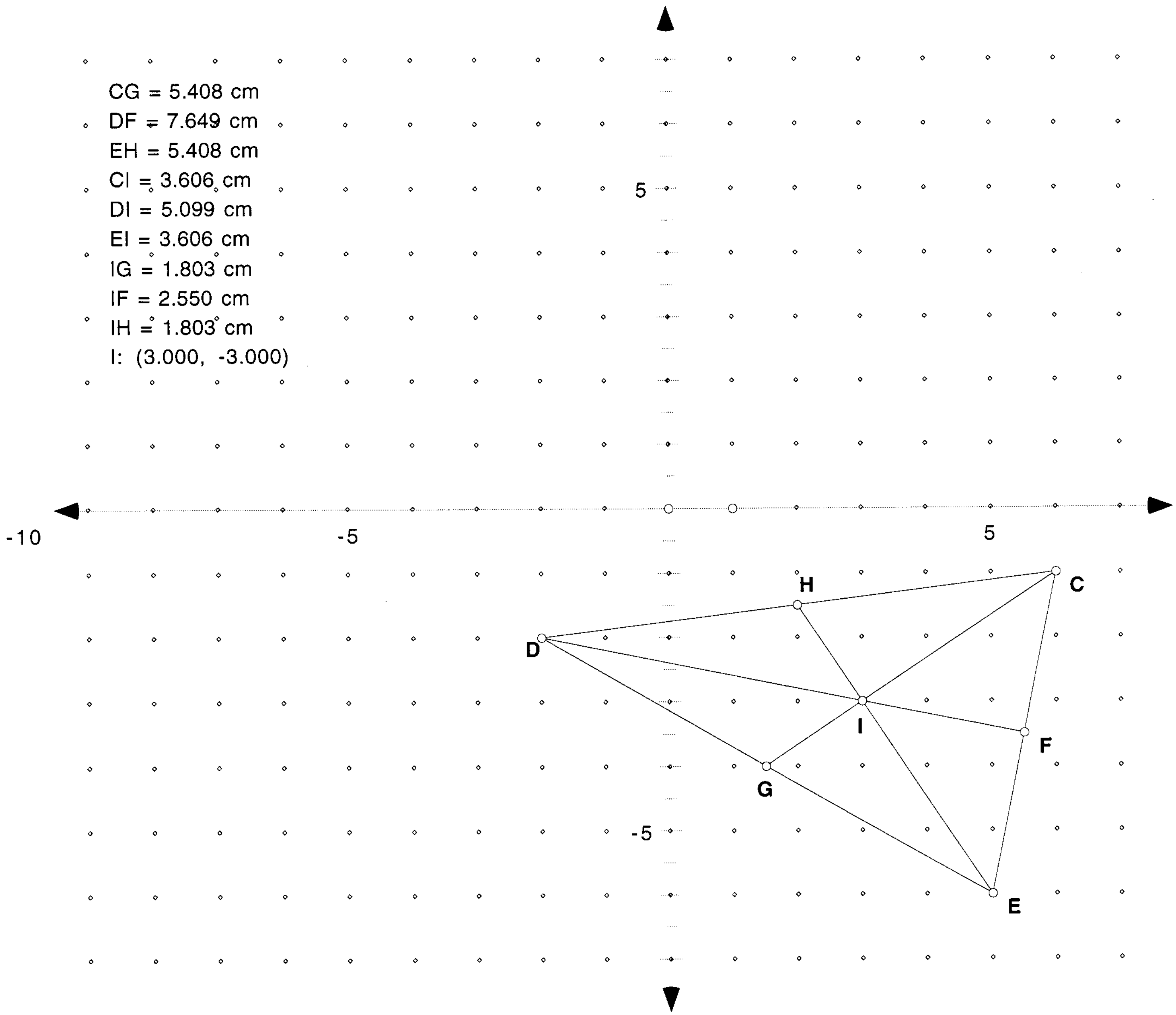
DIRECTIONS: Using the graph menu in Geometer's Sketchpad (or a piece of graph paper) plot the following points $C(6,-1)$, $D(-2,-2)$, and $E(5,-6)$ as the vertices of a triangle. Label the midpoint of $\overline{CD}$ as H, the midpoint of $\overline{CE}$ as F and the midpoint of $\overline{DE}$ as G. Label the centroid as I. Determine the measurements of all the requested segments. If you use Geometer's Sketchpad to construct the triangle, you must print out the screen on which your work is done and include it with this assessment.. If you use a piece of graph paper, you are to include it when you turn in the assessment

Refer to your triangle to complete the following. (If possible, give the segment measures accurate to the thousandths place)

1. CG = ___***5.408***__ inches
2. DF = ___***7.649***__ inches
3. EH = ___***5.408***__ inches
4. CI = ____***3.606***__ inches
5. DI = ____***5.099***__ inches
6. EI = __***3.606***___ inches
7. IG = __***1.803***___ inches
8. IF = __***2.550***___ inches
9. IH = __***1.803***___ inches
10. Name the coordinates for the centroid. __***(3,-3)***____
11. Explain the special relationship that exists between the length of the median and the location of the centroid. Support your statements by using examples from the triangle and its values above.

    The centroid is located two-thirds of the way from the vertex to the opposite side on each of the medians. That means that the ratio of the segments from vertex to centroid with centroid to midpoint is 2:1 For example, in the assessment triangle above, CI= 3.606 inches and CG = 5.408 inches (other pairings are equally good) and CI:IG = 3.606:1.803 which is a 2:1 ratio. The coordinates of the centroid are the averages of the coordinates of the vertices of the triangle, that is the x-coordinate of the centroid can be found by taking (6+-2+5)/3 = 3, and the y-coordinate is found by computing (-1+-2+-6)/3 = -3.

    (A sample of a typical student screen from Geometer's Sketchpad also is attached.)


CG = 5.408 cm
DF = 7.649 cm
EH = 5.408 cm
CI = 3.606 cm
DI = 5.099 cm
EI = 3.606 cm
IG = 1.803 cm
IF = 2.550 cm
IH = 1.803 cm
I: (3.000, -3.000)
5
-10
-5
5
-5
H
C
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E